

Excel based finite difference modeling of ground water flow

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Abstract

This paper presents a simple ground water modeling spreadsheet template developed in Excel with macros written in Visual Basic for Applications (VBA). It is based on Finite Difference method and can be used to model an aquifer of any shape and size. The given template can be expanded to build ground water flow models of virtually unlimited number of nodes. The final modeled heads, at each node, can be output as XYZ file, which can be girded by any contouring software to produce contour maps and 3D surfaces.

1. Introduction

Several types of models have been used to study groundwater flow system. These can be divided into three broad categories analog model, mathematical models and analytical and numerical models (Prickett, 1975). Hydrogeological studies usually involve mathematical modeling of ground water flow. Such models consist of a set of differential equations which govern the flow of groundwater. They have been in use since the late 1800's, but have found widespread application with the increase in available computing power. In computer programming these models are implemented using different approaches among which the finite difference and finite element methods are most common (Wang and Anderson, 1982). In both these methods a system of nodal points is superimposed over the problem domain. The difference between the two methods is in the distribution of nodes. The finite difference nodes are in a regular grid order where nodes can be block-centered or mesh-centered. The finite element methods, on the other hand, can have an irregular distribution of nodes which are connected together to form triangular sub-areas called elements.

Several commercial software applications for groundwater modeling exist (Anderson and Woessner, 1992). To use these applications properly, the user needs to be fully trained. In this paper we present an Excel spreadsheet template for modeling groundwater which is very easy to use and requires no training. Due to the grid nature of Excel cells the finite difference method is used where each cell represents a grid node. Pre-programmed sample cells for interior nodes and no-flow boundary nodes for different sides and corners are given in the template. These cells are copied to the design region according to the requirements of the aquifer to be modeled. Boundary values for constant head nodes are defined and the iterative procedure is initiated to get the flow model. This spreadsheet

template can be used in teaching as well as real groundwater modeling projects.

2. Mathematical background

The physics of groundwater flow in three dimensions is defined by Darcy's law as

$$q_x = -K \frac{\partial h}{\partial x} ; q_y = -K \frac{\partial h}{\partial y} ; q_z = -K \frac{\partial h}{\partial z} \dots\dots\dots(1)$$

where q_x, q_y, q_z are the specific discharge in the x, y, z directions, K is the Hydraulic Conductivity, and h is the head which is the function of all three space coordinates and therefore represented by partial derivatives.

The second important law is the continuity or conservation which for steady state conditions states that the amount of water flowing into a representative elemental volume must be equal to the amount flowing out. Mathematically it can be expressed by the continuity equation as

$$\frac{\partial q_x}{\partial x} + \frac{\partial q_y}{\partial y} + \frac{\partial q_z}{\partial z} = 0 \dots\dots\dots(2)$$

Now combining the above two equations and assuming K to be independent of x, y, z for a homogeneous and isotropic

The finite difference approximation to Laplace's equation (McDonald and Harbaugh, 1998) for such a grid is given by aquifer we get a single second-order partial differential equation

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0 \dots\dots\dots(3)$$

This is Laplace's equation which governs the flow of groundwater through an isotropic, homogeneous aquifer under steady state conditions. This equation simply states that the sum of partial derivatives of head (h) with respect to x, y, and z is zero. The solution of Laplace's equation requires specification of boundary conditions such as Dirichlet conditions and Neumann conditions (Neuman, 1973). As currently we are dealing with ground water flow in two dimensions therefore the Laplace's equation reduces to

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} = 0 \quad \dots\dots\dots(4)$$

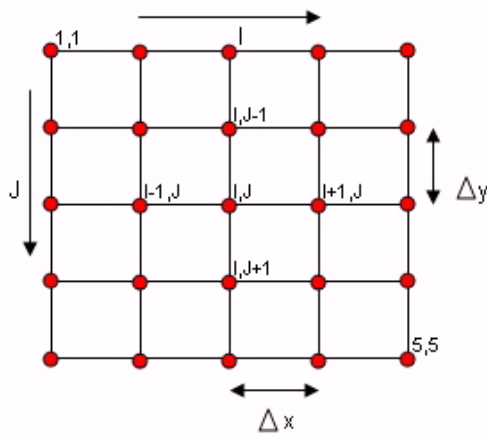


Fig. 1. Finite difference grid of nodes.

Now consider a regularly spaced grid of nodes represented by *i* columns and *j* rows and horizontally and vertically spaced by distances Δx and Δy respectively as shown in figure 1. The head at node *i, j* is $h_{i,j}$.

$$\frac{\partial^2 h}{\partial x^2} \approx \frac{h_{i-1,j} - 2h_{i,j} + h_{i+1,j}}{(\Delta x)^2}$$

$$\frac{\partial^2 h}{\partial y^2} \approx \frac{h_{i,j-1} - 2h_{i,j} + h_{i,j+1}}{(\Delta y)^2}$$

Adding these two terms according to Laplace's equation and considering $\Delta x = \Delta y$ for a square grid we have

$$h_{i-1,j} + h_{i+1,j} + h_{i,j-1} + h_{i,j+1} - 4h_{i,j} = 0 \quad \dots(5)$$

The above equation is the most widely used equation in finite difference solutions of steady-state flow problems and forms the heart of this program. It is iteratively used in the form of a kernel or Laplacian operator for computation of heads at each node in the grid. Such operators are also used in satellite image processing for edge detection and image smoothing (El-Sheimy et al., 2005).

3. Working procedure

The complete working procedure of the presented modeling template is given in Figure 2. Full details for using this template are given as a help worksheet in the Excel Modeling file. To design a new model the user simply needs to copy and paste the required sample cells into the design region of the template (Fig. 3) according to the shape, size and type of boundaries that define the aquifer. The values for constant head nodes at the boundaries are also defined.

If required the iteration parameters; Maximum iterations (default 100) and Maximum change (default .001) are set using the Excel menu; Tools > Options > Calculation Tab. During each iteration the value of heads at all nodes are calculated. The program keeps on iterating unless convergence occurs i.e. the absolute difference in heads at each node from previous and current iteration is less than the Maximum change value.

This parameter is often critical to the modeling process. Too large a value will degrade the final solution, because convergence will not really have occurred. Too small a value will lead to very long execution times and often result in no convergence because computational errors may cause the difference in heads to remain larger than the Maximum change parameter. For this purpose the Maximum iteration value is also given so that the program does not enter into an infinite iterative loop. Thus iterations stop when conditions for either of these two parameters are satisfied. Now to start the iterative process the user simply needs to press F9 key. The values in each cell representing a grid node will keep on changing unless convergence occurs and a steady state flow is attained.

The modeled head values for the grid nodes can be saved as a XYZ file (X, Y are columns, rows and Z is head) using the Visual Basic for Applications macro. It is activated by pressing Ctrl-S keys for entering the output filename. The output file can be used by Surfer or any other contouring software for girding and subsequent contouring or surface generation. The final contour map represents the variation of head in the modeled region, where flow lines are considered to be moving perpendicular to the contour lines from higher to lower potential.

Simulation results of this spreadsheet can be utilized in different commercial software's like PROCESSING MODFLOW (Chiang and Kinzelbach, 1998) etc. to develop a complete numerical ground water model.

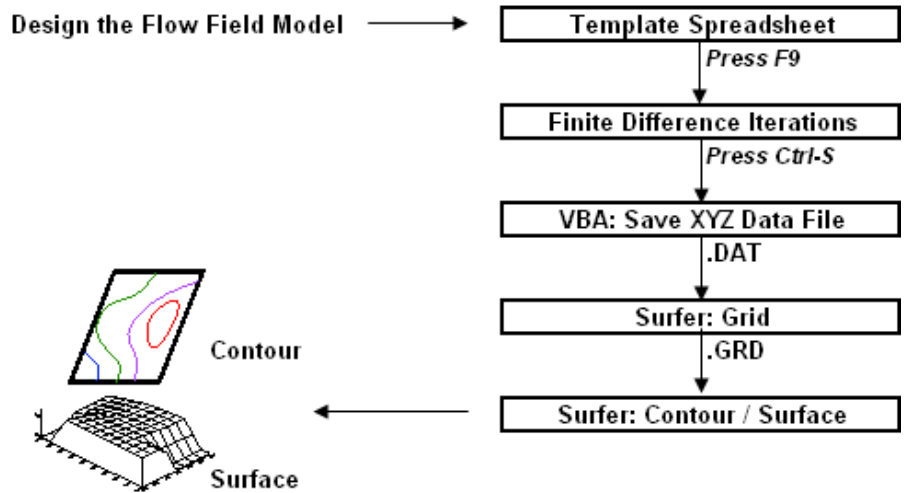


Fig. 2. Processing flow chart of the modeling template along with Surfer.

Normal	Left Side		Right Side		Top Side		Bottom Side		Top-Left		Top-Right		Bottom-Left		Bottom-Right					
Interior Node	No-flow		No-flow		No-flow		No-flow		No-flow		No-flow		No-flow		No-flow					
	1		1					1						1		1				
1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1	1				
	1					1				1										
	x Exterior Node																			
	A 1	B 2	C 3	D 4	E 5	F 6	G 7	H 8	I 9	J 10	K 11	L 12	M 13	N 14	O 15	P 16	Q 17	R 18	S 19	T 20
1																				
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Fig. 3. Excel spreadsheet template for modeling groundwater. Cells (gray to dark gray) at the top are pre-programmed for interior nodes and various types of no-flow boundary nodes. The empty cells below form the design region where the model is created.

4. Practical examples

To demonstrate the design, working and output of the modeling spreadsheet template two examples are shown in Figures 4 and 5. They also show the final graphical outputs generated by contouring software. These examples are also included in the Excel Modeling file as example worksheets. They help the user in designing their own groundwater models in the empty spreadsheet template.

5. Conclusions

It is concluded that Excel based spreadsheet modeling can be effectively utilized to develop the equipotential surface features of the groundwater flow regimes. Directions of groundwater flows are monitored by assigning the vector lines at right angle to these equipotential surfaces. Groundwater volume in active storage can be ascertained from the complete flow-net based on Excel based spreadsheet modeling. In addition it can also be used, along with contouring software, to generate small to large groundwater flow models.

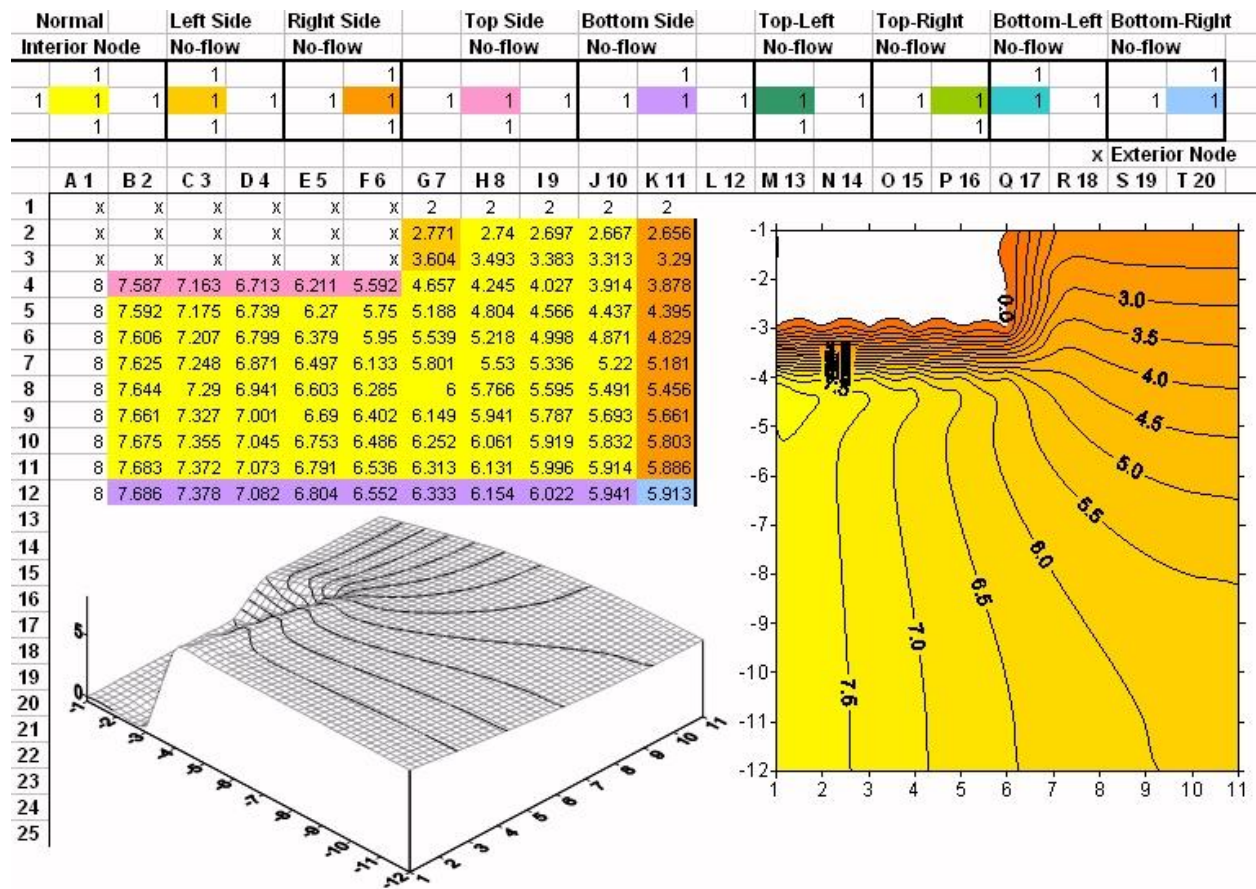


Fig. 4. Example of a small groundwater flow model with contour map and 3D surface from Surfer.

